Exercise 20

If $\tanh x = \frac{12}{13}$, find the values of the other hyperbolic functions at x.

Solution

Suppose that $\tanh x = \frac{12}{13}$. Then

$$\text{sech}^2 x = 1 - \tanh^2 x = 1 - \left(\frac{12}{13}\right)^2 = \frac{25}{169}.$$

Take the square root of both sides.

$$\text{sech}\, x = \pm\frac{5}{13}$$

Since hyperbolic cosine and hyperbolic secant are always positive, choose the plus sign.

$$\boxed{\text{sech}\, x = \frac{5}{13}}$$

Since $1/\cosh x = \text{sech}\, x$,

$$\boxed{\cosh x = \frac{13}{5}.}$$

Since $\cosh^2 x - \sinh^2 x = 1$,

$$\sinh^2 x = \cosh^2 x - 1 = \left(\frac{13}{5}\right)^2 - 1 = \frac{144}{25}.$$

Take the square root of both sides.

$$\sinh x = \pm\frac{12}{5}$$

Since $\tanh x$ is positive, choose the plus sign.

$$\boxed{\sinh x = \frac{12}{5}}$$

Since $1/\sinh x = \text{csch}\, x$,

$$\boxed{\text{csch}\, x = \frac{5}{12}.}$$

And since $1/\tanh x = \coth x$,

$$\boxed{\coth x = \frac{13}{12}.}$$